Neutrino masses and lepton-number violation
in the Littlest Higgs scenario

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Abstract

We investigate the sources of neutrino mass generation in Little Higgs theories, by confining ourselves to the "Littlest Higgs" scenario. Our conclusion is that the most satisfactory way of incorporating neutrino masses is to include a lepton-number violating interaction between the scalar triplet and lepton doublets. The tree-level neutrino masses generated by the vacuum expectation value of the triplet are found to dominate over contributions from dimension-five operators so long as no additional large lepton-number violating physics exists at the cut-off scale of the effective theory. We also calculate the various decay branching ratios of the charged and neutral scalar triplet states, in regions of the parameter space consistent with the observed neutrino masses, hoping to search for signals of lepton-number violating interactions in collider experiments.

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I.   INTRODUCTION

Little Higgs theories [1, 2, 3] represent a new attempt to address the problem of quadratic divergence in the mass of the Higgs boson responsible for electroweak symmetry breaking. This approach treats the Higgs boson as part of an assortment of pseudo-Goldstone bosons, arising from a global symmetry spontaneously broken at an energy scale Λ, typically on the order of 10 TeV. There is also an explicit breakdown of the overseeing global symmetry via gauge and Yukawa interactions, thereby endowing the Goldstone bosons with a Coleman-Weinberg potential and making them massive. The Higgs mass is thus protected by the global symmetries of the theory and only arises radiatively due to the gauge and Yukawa interactions. As an effective theory valid up to the scale Λ, the model is rather economical in terms of the new fields introduced in order to fulfill the necessary cancellation for the quadratic divergence at the one-loop level. The model requires, in addition to new gauge bosons and vectorlike fermions, the existence of additional scalars belonging to certain representations of the Standard Model (SM) gauge group.

Aside from the crucial vector-like T-quark, the fermionic sector can essentially have the same appearance as in the SM. There is no attempt made to address the origin of fermion masses and mixing. In fact, the theory would encounter extremely stringent constraints from the absence of excessively large flavor-changing neutral currents and CP violation in the fermionic sector [4] if the scale responsible for flavor physics is at the order of the cutoff scale Λ. Flavor issues are thus ostensibly left out as problems awaiting the more fundamental theory at higher energies, the so-called UV completion of the theory, that would hopefully lead to the SM structure or similar as an effective low-energy realization.

However, one may like to remember that the only area where experimental hints of new physics have been found so far is the neutrino sector [5]. It is therefore both interesting and important to see if little Higgs theories can accommodate neutrino masses and mixing as suggested by the observed data. It is even not unreasonable to say that it will be a vindication of little Higgs theories if they at least suggest a mechanism for the generation of neutrino masses. The present work aims to buttress this attempt. Are the neutrinos acquiring their masses through interaction with new particles already postulated in the theory? What can be the detectable signatures of the model carrying imprints of the fact that its low-energy Lagrangian and particle spectrum address the issue of neutrino masses?

We examine these questions by adopting the "Littlest Higgs" (LtH) model [2], which has been extensively studied in recent literature.

We explore the most economic extension of the basic model that is required to accommodate neutrino masses and is consistent with the demand that it does not affect the cancellation of quadratic divergences in the SM Higgs mass. In particular, we make use of the fact that the LtH scenario contains, in addition to the usual Higgs doublet, an additional set of scalars that form a complex triplet [6] under the SU(2) gauge group of the Standard Model with hypercharge $Y = 1$ ($Q = I_3 + Y$). This complex triplet forms part of the assortment of Goldstone bosons when a global SU(5) breaks down to SO(5) at the scale Λ in this model. There is an additional gauged SU(2)×U(1) beyond that of the SM, which is also spontaneously broken at scale Λ; some of the aforementioned Goldstone bosons are absorbed as longitudinal components of the extra gauge bosons. Ten scalar degrees of freedom remain after this, and are found to consist of a doublet (H) and a complex triplet (ϕ) under the electroweak SU(2). The complex triplet offers a chance to introduce lepton number violating interactions into the theory. We find that the most satisfactory way of incorporating neutrino masses is to exploit such an interaction of the lepton doublets, leading to a Majorana mass for neutrinos and lepton number violation by two units. Then we proceed to examine the parameter range of this model consistent with the observed neutrino masses, and look at the consequence it has on the phenomenology of the model. In particular, we focus on the decays of the additional SU(2) triplet scalar states introduced in this scenario, which can have masses of order a TeV. We present calculations of the decay branching ratios of the triplet states, discuss the complementary roles of different decay channels to test the scenario, and comment on their potential collider signatures within the region of the parameter space that is consistent with the observed neutrino masses.

Our paper is organized as follows. In Section II, the status of neutrino mass generation with a heavy right-handed neutrino is first briefly reviewed. We then take up the case of neutrino masses without any right-handed neutrino, and show that the LtH construction can accommodate the observed neutrino mass and mixing patterns. In particular, with the help of the complex triplet, one obtains dimension-4 lepton-number violating operators ($\Delta L = 2$). The Majorana neutrino masses and their mixing can be generated by these operators consistent with current observations without necessarily pushing the couplings to tiny values; instead, the smallness of the neutrino masses can be driven in part by a tiny

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triplet vev. We also discuss the $\Delta L = 2$ operators with the full gauge symmetry of the model and find that in such a scenario the couplings would have to be of order 10^{-11} to accommodate the observed neutrino masses. In Section III, we study the decay channels of the triplets. These, we emphasize, constitute the characteristic signals of the triplet and allow a test of the mechanism of neutrino mass generation. We summarize and conclude in Section IV. The features of the LtH scenario and the interactions of the triplet that are relevant for our phenomenological study are summarized in Appendix A. The triplet decay partial widths are listed in Appendix B.

II. NEUTRINO MASSES

A. Neutrino masses with right-handed neutrinos

In the SM as well as the simplest little Higgs constructions, there are no right-handed neutrino states that are singlets under SM gauge interactions. By introducing right-handed neutrinos (N_R), one can obtain gauge-invariant Dirac mass terms from the SU(2) doublets of the leptons L and the Higgs H,

$$y_{ij}^D \, \overline{L_{Li}} \, H^\dagger N_{Rj} + \text{h.c.}, \tag{1}$$

with i, j being generation indices, as well as Majorana mass terms

$$- M_{ij}\overline{N_{Ri}^c}N_{Rj} + \text{h.c.} = M_{ij}N_{Ri}^T C^{-1}N_{Rj} + \text{h.c.}, \tag{2}$$

where C is the charge-conjugation operator in the notation of, e.g., Ref. [7].

The Dirac terms alone lead to a contribution to the neutrino mass of the order $m_\nu \sim y^D v$. Since the neutrino masses are known to be at most of order 0.3 eV [8], the Yukawa couplings would have to be extremely small, $y^D \lesssim 10^{-12}$. While technically natural, such tiny Yukawa couplings are difficult to rationalize.

Including the Majorana terms, light neutrino masses are generated at the order $(y^D v)^2/M$ [9] by virtue of the well-known seesaw mechanism [10]. If we assume that the Yukawa couplings y_{ij}^D are naturally of the order of unity, then $M \gtrsim 10^{13}$ GeV in order to obtain a neutrino mass less than about 0.3 eV. The problem, however, is that if we take the Majorana mass scale to be near the Little Higgs cutoff $\Lambda \simeq 10$ TeV, then all of the Yukawa couplings

would have to be quite small and all roughly equal, $y_{ij}^D \lesssim 10^{-5}$ for all three generations. This is in constrast to the corresponding charged leptons, for which the Yukawa couplings exhibit a large hierarchy between generations. Of course, the right-handed neutrino mass that determines the seesaw scale could be much higher than Λ, as in the usual seesaw scenario within the Standard Model. However, in this work we wish to look for alternative explanations of the neutrino masses within the context of the LtH scenario with observable signatures that do not rely upon physics above the cutoff scale Λ.

B. Neutrino masses in the absence of right-handed neutrinos

To us, the solution seems to be in avoiding the introduction of massive right-handed neutrinos altogether in a little Higgs scenario. One can still construct Majorana mass terms with the help of the Higgs triplet in the LtH model, obtained from a dimension-four $\Delta L = 2$ coupling,

$$\mathcal{L} = iY_{ij}L_i^T \ \phi \, C^{-1}L_j + \text{h.c.} \tag{3}$$

Note that the definition of ϕ here includes $(-i)$, as evident from Eq. (A3). With the vacuum expectation value (vev) of ϕ^0 being v', the induced neutrino masses are of the order of Yv'. With a sufficiently small v', as preferred by the precision electroweak data [11], adequate neutrino masses may be generated. The occurrence of such Majorana masses has already been discussed in the context of general models with triplet scalars [6, 12].

In the LtH model, however, some additional caution is necessary, since here we have an effective theory with a rather low cut-off. It can be argued that, if there is lepton-number violating physics at the scale Λ, then it is practically impossible to prevent the appearance of dimension-five operators of the form

$$Y_5\frac{(HL)^2}{\Lambda} \tag{4}$$

giving rise to neutrino masses on the order of Y_5v^2/Λ. This contribution to the neutrino masses is inadmissibly large if Y_5 is naturally of order unity. Of course, one may suppress the neutrino mass by requiring that the seesaw scale corresponding to lepton-number violation is not Λ (~ 10 TeV) but some higher scale, perhaps corresponding to a grand unification scenario. However, as we have mentioned above, this solution is somewhat unsatisfying in the

little Higgs context, since the entire issue of grand unification is unclear in a UV-incomplete theory.

The way out of the difficulty is to postulate that there is *no additional lepton-number violating physics at the scale* Λ, and that the *only* $\Delta L = 2$ effect comes from the coupling given by Eq. (3). Such a postulate is plausible in the sense that the operator of Eq. (3) is renormalizable and independent of the cutoff. Such a postulate also keeps the scenario minimal in terms of particle content, since right-handed neutrinos, unlike the scalar triplets, do not arise from any intrinsic requirement of the model. The absence of right-handed neutrinos at or below the scale Λ prevents the potentially dominant dimension-five operators of Eq. (4). Such operators can then arise only through loop effects involving the $\Delta L = 2$ couplings of the ν_L to the scalar triplet. As we demonstrate below in Sec. II C, the structure of the Coleman-Weinberg potential ensures that the contributions of these loop-induced dimension-five operators to the neutrino masses are subleading compared to the tree-level $\Delta L = 2$ interaction given above.

Thus, neutrino masses are perhaps best implemented in the LtH model solely in terms of the tree-level $\Delta L = 2$ interaction of the scalar triplet. So far there is no need to attribute the effect to a high scale, since lepton-number conservation is not dictated by any underlying symmetry of the theory. The relevance of this term is further accentuated by the fact that the triplet vev in any case has to be quite small compared to the electroweak scale, in order to be consistent with the limits on the ρ-parameter [11, 13]. Thus, seeds of small neutrino masses can already be linked to the electroweak precision constraints.

It should be noted that although the $LL\phi$ interaction term is invariant under the standard SU(2)$_L$×U(1)$_Y$ symmetry, it breaks the full [SU(2)×U(1)]^2 gauge invariance of the LtH model. The $LL\phi$ interaction term is invariant under the two U(1) symmetries so long as the U(1) charges of the lepton doublet are chosen to cancel anomalies in the full theory (see Sec. II D for details). On the other hand, this term breaks the [SU(2)]^2 part of the full gauge symmetry because the triplet ϕ is a Goldstone boson of the full theory and transforms nonlinearly under the two SU(2)s, while L transforms as a doublet under only SU(2)$_1$. We note however that the real motivation for this enlarged gauge symmetry is the cancellation of potentially large quadratically divergent contributions to the Higgs mass. Apart from that, there is no requirement that such an invariance holds in all sectors of the theory.

It can be seen through explicit calculation that the cancellation of quadratic divergences is

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not affected so long as the non-invariance under $[\text{SU}(2)\times\text{U}(1)]^2$ is confined only to the lepton-number violating interaction of the triplet. In particular, the global symmetry structure in the gauge and top-quark sectors that protects the Higgs mass at one-loop level is not affected by the new $LL\phi$ interaction. The only effect of this interaction on the Coleman-Weinberg potential [14] for the scalars is a contribution to the coefficient of the triplet mass, λ_{ϕ^2} (see Appendix A for details), for which the modified one-loop expression is

$$\lambda_{\phi^2} = \frac{a}{2}\left[\frac{g^2}{s^2 c^2} + \frac{g'^2}{s'^2 c'^2}\right] + 8a'\lambda_1^2 + a''\text{Tr}(YY^\dagger), \tag{5}$$

where a'' is an arbitrary $\mathcal{O}(1)$ constant reflecting the UV incomplete nature of the theory. The overall constraint to be satisfied by the modified expression is that λ_{ϕ^2} should remain positive, so that the triplet vev, purportedly small, is generated through doublet-triplet mixing only. If a'' is positive, it results in a slight enhancement of the triplet scalar mass compared to that in the minimal LtH scenario. Thus the introduction of the $LL\phi$ interaction seems to be consistent with the fundamental spirit of the little Higgs approach. We lay out the full interaction terms of Eq. (3) in Appendix A for future phenomenological considerations.

C. Constraints from neutrino masses

Since our first concern is to see the viability of this proposal, we begin by assuming neutrino masses to be of order 0.1 eV. The left-handed Majorana neutrino mass matrix resulting from Eq. (3) in this scenario is

$$\mathcal{M}_{ij} = Y_{ij}v'. \tag{6}$$

We neglect CP-violating phases. Then Y is a (3×3) symmetric matrix with six independent parameters. The physical neutrino masses are the product of v' and the eigenvalues of Y. The elements of Y can in principle be as large as perturbation theory permits; we consider them to have a natural size of order unity. The triplet vev v' is restricted to be $\lesssim 1$ GeV from the constraints on the ρ-parameter [11, 13].

The smallness of the neutrino masses leaves us with two extreme alternatives, as described below.

1. The elements of Y are very small, typically of the order 10^{-10}, and $v' \sim 1$ GeV. This means that the $LL\phi$ interaction term in Eq. (3) supplies the physics responsible for the smallness of neutrino masses.

2. $Y \simeq 1$ together with an extremely small v', arising from a tiny value of the induced doublet-triplet mixing coefficient $\lambda_{h\phi h}$ in the Coleman-Weinberg potential. In this case the Coleman-Weinberg potential provides the physics behind the smallness of neutruino masses, while the origin of bi-large mixing has to be sought in the relative values of the different Y_{ij}.

The first option leads to very small couplings, which could be argued to be unnatural. One needs to remember that the physics linked with Y_{ij} is *not only lepton-number violation but also lepton-flavor violation*. Therefore the coupling in Eq. (3) must have its origin at a scale much higher than Λ, in order to avoid unacceptable flavor violation in the low-energy theory and the appearance of large dimension-five operators. Thus the explanation for the smallness of the neutrino masses is pushed up to scales much higher than Λ.

The second scenario, on the other hand, has a certain advantage. In addition to generating neutrino masses of the right order, one also has to explain the observed bi-large mixing pattern in the neutrino sector. A model-independent fit of such mixing requires one to fine-tune the elements of Y. Having all six elements on the order of 10^{-10} enhances the degree of fine-tuning even further. It may therefore be a slightly less disquieting prospect to envision the "fine-tuned" elements of Y as being close to unity, and have a very small vev for the triplet. The generation of such a small vev must be accomplished by appropriate values of the parameters that determine $\lambda_{h\phi h}$ at the scale Λ. As can be seen from the detailed expressions listed in Appendix A 1, a small triplet vev can arise, for example, from a cancellation of the gauge and Yukawa contributions to the Coleman-Weinberg potential. While a theoretical explanation has to await the UV completion of the scenario, this situation is consistent with all other aspects of the model, and has distinctive phenomenological implications. Thus we have chosen to explore such implications in detail, remembering all along that the final explanation for the smallness of the neutrino masses is linked to the UV completion of the LtH model.

To summarize, we will concentrate only on the operator of Eq. (3), with the requirement $m_\nu \simeq Y v' \simeq 10^{-10}$ GeV. Within this constraint, the $\Delta L = 2$ coupling Y [which is actually
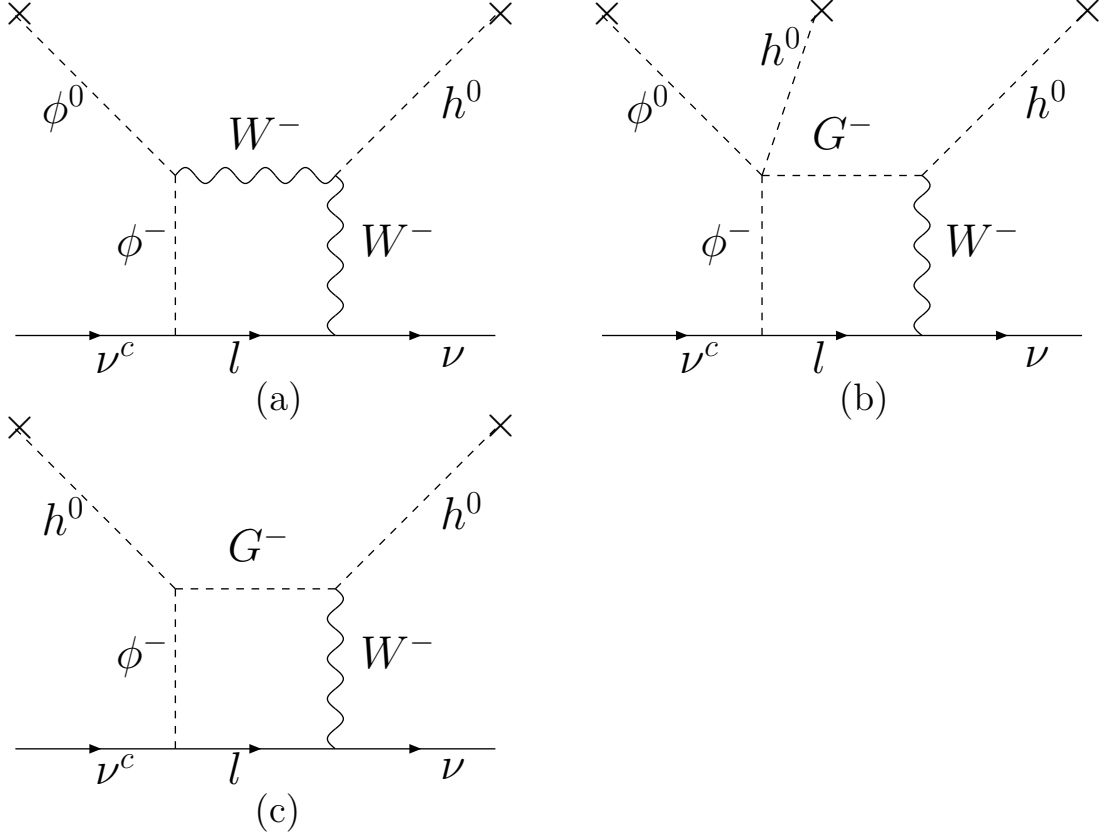
FIG. 1: Representative one-loop diagrams giving rise to neutrino masses via dimension-five operators.

a (3×3) matrix] and the triplet vev v' can vary over a wide range in our formulation. As we shall see in the next section, the phenomenological consequences are especially interesting in the parameter ranges

$$10^{-5} < Y_{ij} \lesssim 1, \qquad 0.1 \text{ MeV} > v' > 1 \text{ eV}. \tag{7}$$

It is important not to overlook other potentially significant contributions to the neutrino masses through dimension-five operators induced at the one-loop level. Some representative diagrams leading to such operators are shown in Fig. 1, where we have worked in the 't Hooft-Feynman gauge. All of these diagrams give neutrino masses of the form $M_{ij}\nu_{Li}^T C^{-1}\nu_{Lj}$.

The neutrino mass from Fig. 1(a) is

$$\mathcal{M}_{ij} = ivv' \frac{M_W g^3 Y_{ij}}{\sqrt{2}} \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m_\phi^2)(p^2 - M_W^2)^2} \approx (Y_{ij}v') \frac{g^4 \ v^2}{32\sqrt{2}\pi^2 m_\phi^2}. \tag{8}$$

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Clearly this is a subleading contribution compared to $Y_{ij}v'$, being suppressed by a loop factor times v^2/m_ϕ^2. Similarly, the contribution from Fig. 1(b) is

$$\mathcal{M}_{ij} \approx -(Y_{ij}v')\frac{g^2\lambda_{h\phi\phi h}v^2}{32\sqrt{2}\pi^2 m_\phi^2} = (Y_{ij}v')\frac{g^2 v^2}{24\sqrt{2}\pi^2 f^2}, \tag{9}$$

where we have used the relation $\lambda_{h\phi\phi h} = -4\lambda_{\phi^2}/3 = -4m_\phi^2/3f^2$ (see Appendix A for details). This is again suppressed by a loop factor times v^2/f^2.

The contribution from Fig. 1(c) is

$$\mathcal{M}_{ij} = i\frac{g^2}{4}\frac{Y_{ij}}{\sqrt{2}}\lambda_{h\phi h}fv^2\int\frac{d^4 p}{(2\pi)^4}\frac{1}{(p^2 - m_\phi^2)(p^2 - M_W^2)^2} \approx (Y_{ij}v')\frac{g^2}{32\sqrt{2}\pi^2}, \tag{10}$$

where we have made use of the relation $v' = \lambda_{h\phi h}v^2/2\lambda_{\phi^2}f$ from the minimization conditions of the Coleman-Weinberg potential and $\lambda_{\phi^2}f^2 \simeq m_\phi^2$ (see Appendix A). Unlike the other diagrams, the neutrino mass contribution from this diagram is suppressed by only a loop factor. This is bacause the size of the diagram in Fig. 1(c) is controlled by the doublet vev v. The special relationships among parameters in the Coleman-Weinberg potential enables one to re-express the contribution in terms of the triplet vev v'. The contribution is larger than that from Figs. 1(a) and (b), although the loop factor ensures that it is subleading. Such a contribution can play a potentially important role in determining the precise values of the neutrino mixing angles.

There is also a diagram in which the vertical W^- propagator in Fig. 1(a) is replaced by the charged Goldstone G^-. Since the coupling of G^- to $\bar{\nu}, \ell$ is suppressed by m_ℓ/M_W, this diagram gives only a small contribution. Similarly, diagrams involving a virtual Z boson and neutrinos are negligible. Finally, the above expressions are subject to additional corrections due to doublet-triplet mixing, which are further suppressed by v'/v.

D. $\Delta L = 2$ operators with larger symmetry

Thus far, our approach to constructing lepton-number violating operators has been guided by the SM gauge invariance and naturalness considerations, subject to the experimental constraints on the neutrino masses and mixing patterns. The treatment of the scalar triplet separate from the doublet requires some mechanism to split the interactions of these two components of the non-linear Σ field, which is beyond the scope of our phenomenological considerations in the current work. Nevertheless, it is tempting to ask if one can in-

	$\Sigma^*_{\alpha\beta}$	L	$L^T_\alpha \Sigma^*_{\alpha\beta} C^{-1} L_\beta$
U(1)$_1$	3/5	$3/10 - y_e$	$6/5 - 2y_e$
U(1)$_2$	2/5	$-4/5 + y_e$	$-6/5 + 2y_e$
Hypercharge	1	$-1/2$	0

TABLE I: Charge assignments of the lepton and scalar fields and of the operator in Eq. (11) under the two U(1) gauge groups and hypercharge, with $\alpha, \beta = 1, 2$.

stead construct operators that respect the full gauge symmetry of the LtH model, namely [SU(2)×U(1)]^2 gauge invariance.

Following the conventions of Refs. [2, 15], in which the third-generation quark doublet is extended to $\chi^T = (b_L\ t_L\ T_L)$, we write the lepton doublets as $L^T = (\ell_L\ \nu_L)$. We can then write down the following lepton flavor violating operator,

$$\mathcal{L}_{LFV} = -\frac{1}{2} Y_{ij} f \left(L^T_i \right)_\alpha \Sigma^*_{\alpha\beta} C^{-1} \left(L_j \right)_\beta + \text{h.c.}, \tag{11}$$

where i, j are generation indices and $\alpha, \beta = 1, 2$ are SU(5) indices. This operator is gauge invariant under both the SU(2)$_{1,2}$ gauge groups and under hypercharge. This operator is also gauge invariant under both of the U(1)$_{1,2}$ gauge groups if the lepton charges under the two U(1) groups are given by $Y_1(L) = -3/10$ and $Y_2(L) = -1/5$. In the notation of Ref. [15], this corresponds to $y_e = 3/5$, as shown in Table I. This is the same condition that ensures anomaly cancellation among the SM fermions. This can be understood as follows. The anomaly cancellation condition is satisfied when the U(1)$_{1,2}$ charges of the fermions are proportional to their hypercharges. Since the operator in Eq. (11) conserves hypercharge, the anomaly-free condition is sufficient to ensure that this operator also conserves the U(1)$_{1,2}$ charges.

Expanding the upper two-by-two block of the matrix $\Sigma^*_{\alpha\beta}$ in terms of the scalar fields H and ϕ (see Appendix A), we have

$$\Sigma^*_{\alpha\beta} = -\frac{2}{f} \begin{pmatrix} \phi^{++} & \phi^+/\sqrt{2} \\ \phi^+/\sqrt{2} & \phi^0 \end{pmatrix} - \frac{1}{f^2} \begin{pmatrix} h^+h^+ & h^+h^0 \\ h^+h^0 & h^0h^0 \end{pmatrix} + \cdots \tag{12}$$

Inserting this into Eq. (11), we obtain

$$\mathcal{L}_{LFV} = Y_{ij} \left[\nu^T_{Li} C^{-1} \nu_{Lj} \left(\phi^0 + \frac{1}{2f} h^0 h^0 \right) + \left(\nu^T_{Li} C^{-1} \ell_{Lj} + \ell^T_{Li} C^{-1} \nu_{Lj} \right) \left(\frac{1}{\sqrt{2}} \phi^+ + \frac{1}{2f} h^+ h^0 \right) \right. $$

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$$+\ell_{Li}^T C^{-1} \ell_{Lj}\left(\phi^{++} + \frac{1}{2f} h^+ h^+\right)\Bigg] + \text{h.c.} \tag{13}$$

Clearly, the nonlinear sigma model has served to relate the dimension-four $\overline{\nu_i^c}\nu_j\phi^0$ coupling to the dimension-five $\overline{\nu_i^c}\nu_j h^0 h^0$ coupling. This gives rise to a mass matrix for the neutrinos involving both v' and v:

$$\mathcal{M}_{ij} = Y_{ij}\left(v' + \frac{v^2}{4f}\right). \tag{14}$$

Equation (13) gives, to the leading order, the following dimension-four couplings of scalars to left-handed lepton pairs:

$$
\begin{aligned}
\mathcal{L}_{LFV}^{dim-4} &= Y_{ij}\left\{\ell_{Li}^T C^{-1}\ell_{Lj}\phi^{++} + \frac{1}{\sqrt{2}}\left(\nu_{Li}^T C^{-1}\ell_{Lj} + \ell_{Li}^T C^{-1}\nu_{Lj}\right)\phi^+ + \nu_{Li}^T C^{-1}\nu_{Lj}\phi^0\right\} \\
&\quad + Y_{ij}^*\left\{\overline{\ell_{Li}}\,C\,\overline{\ell_{Lj}}^T \phi^{--} + \frac{1}{\sqrt{2}}\left(\overline{\nu_{Li}}\,C\,\overline{\ell_{Lj}}^T + \overline{\ell_{Li}}\,C\,\overline{\nu_{Lj}}^T\right)\phi^- + \overline{\nu_{Li}}\,C\,\overline{\nu_{Lj}}^T \phi^{0*}\right\}, \tag{15}
\end{aligned}
$$

where in the second line we have explicitly written out the Hermitian conjugate piece. Note that ϕ^0 is a complex field containing real scalar and pseudoscalar degrees of freedom, $\phi^0 = (\phi^s + i\phi^p)/\sqrt{2}$.

The expression (11) is invariant under the full $[\text{SU}(2)\times\text{U}(1)]^2$ gauge symmetry and preserves the nonlinear sigma model form for the scalar interactions. However, the price to pay in such an approach is that one has to include dimension-5 terms proportional to H^2 from the beginning, and thus have contributions to the neutrino masses proportional to v^2/f. Unlike the dimension-5 operators generated by the diagrams in Fig. 1, these contributions are not proportional to $Y_{ij}v'$ times a loop suppression factor and cannot in general be made small, since $f \simeq \text{TeV}$ if we have to stabilize the Higgs mass. As a result, this approach almost invariably ends up requiring values

$$Y_{ij} \sim 10^{-11}, \tag{16}$$

for the $\Delta L = 2$ couplings of all i, j. They are indeed unnaturally small. This implies the need for a more fundamental explanation for neutrino masses beyond the effective theory at the scale Λ.

On the other hand, in our approach of separating the lepton-number violating couplings of ϕ and H, one can avoid extreme fine-tuning of the Y_{ij} couplings and at the same time ensure neutrino masses of a size consistent with experimental data. This is because our starting point is the dimension-four renormalizable operator of Eq. (3), as opposed to the

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higher dimensional ones discussed in the alternative approach. Thus our formulation by keeping only the L-violating terms of Eq. (3) is independent of the cut-off. It is admittedly a phenomenological approach, and assumes that, whatever be the mechanism responsible for the breakdown of $[\mathrm{SU}(2)\times\mathrm{U}(1)]^2$ in the L-violating sector, any additional induced term proportional to (v^2/f) is suppressed. We nonetheless feel that this approach is quite general and model-independent, especially because the cancellation of quadratically divergent contributions to the SM Higgs mass remains unaffected, as was discussed in Sec. II B.

We take this opportunity to note that an attempt has been recently made in Refs. [16, 17] similar to the approach of Eq. (11). In Ref. [17], this operator was given in the form

$$\mathcal{L}_{LFV} = z_{ij}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}f\left(L_i^T\right)_\alpha \Sigma_{\beta\gamma}^* C^{-1}\left(L_j^T\right)_\delta + \text{h.c.}, \tag{17}$$

which is equivalent to our result if $L^T = (-\nu \; \ell)$ is used in Eq. (17). The authors of Refs. [16, 17] also found the same conclusion as in Eq. (16), that $Y_{ij} \sim 10^{-11}$.

III.   DECAYS OF THE TRIPLET STATES

We now examine the observable consequences of the scalar triplet having a vev and lepton number violating interactions compatible with the observed neutrino masses. In particular, we consider the decays of the scalar triplet into various characteristic final states, and discuss their observable signals in future collider experiments.

First of all, we note that the mechanism of scalar mass generation through the Coleman-Weinberg mechanism [14] in the LtH model implies that the members of the triplet, ϕ^{++}, ϕ^+, ϕ^s, and ϕ^p (where ϕ^s and ϕ^p are the scalar and pseudoscalar components of ϕ^0), are degenerate at lowest order with a common mass m_ϕ. Their masses are split by electroweak symmetry breaking effects, leading to masses $m_\phi[1 + \mathcal{O}(v^2/m_\phi^2)]$. The mass splittings are thus quite small for $m_\phi \gg M_W$, and we will neglect them in what follows. The relevant interaction terms for the $\Delta L = 2$ processes are given in Table II in Appendix A. The other ϕ couplings conserving the lepton number have been given in Ref. [15]. For completeness, they are also tabulated in Table III in Appendix A. The possible decays of the triplet states are

$$\phi^{++} \rightarrow \ell_i^+\ell_j^+, \quad W^+W^+,$$
$$\phi^+ \rightarrow \ell_i^+\bar{\nu}_{\ell_j}, \quad t\bar{b}, \quad T\bar{b}, \quad W^+Z, \quad W^+h,$$

$$\phi^s \rightarrow \nu_i\nu_j, \quad \bar{\nu}_i\bar{\nu}_j, \quad t\bar{t}, \quad b\bar{b}, \quad t\bar{T} + \bar{t}T, \quad ZZ, \quad hh,$$

$$\phi^p \rightarrow \nu_i\nu_j, \quad \bar{\nu}_i\bar{\nu}_j, \quad t\bar{t}, \quad b\bar{b}, \quad t\bar{T} + \bar{t}T, \quad Zh. \tag{18}$$

The full set of partial decay widths is listed in Appendix B.

To clearly see the interesting physics points, we discuss the partial decay widths for the doubly-charged Higgs boson for $m_\phi \gg M_W$,

$$\Gamma(\phi^{++} \rightarrow \ell_i^+\ell_i^+) = \frac{|Y_{ii}|^2 m_\phi}{8\pi}, \quad \Gamma(\phi^{++} \rightarrow W_L^+W_L^+) \approx \frac{v'^2 m_\phi^3}{2\pi v^4}, \quad \Gamma(\phi^{++} \rightarrow W_T^+W_T^+) \approx \frac{g^4 v'^2}{4\pi m_\phi},$$

where W_L (W_T) stands for the longitudinal (transverse) component of the W boson. We first point out that the $\Delta L = 2$ processes, $\phi^{++} \rightarrow \ell_i^+\ell_j^+$, are all driven by the lepton number violating Yukawa coupling Y_{ij}. These decays to the lepton states will constitute the smoking gun signatures of the scenario proposed by us. The decays into two gauge bosons, on the other hand, depend directly on v', the triplet vev. The m_ϕ factors in the numerator in the decay to the longitudinally-polarized gauge bosons come from the typical enhancement $(m_\phi^2/M_W^2)^2$ over the decay to the transversely-polarized gauge bosons, governed by the Goldstone-boson equivalence theorem. The $W_T^\pm W_T^\pm$ mode with a genuine gauge coupling thus becomes vanishingly small at higher m_ϕ.

The complementarity between the $\ell^\pm\ell^\pm$ and $W^\pm W^\pm$ channels for small and large values of v' is clearly seen in Fig. 2: for $m_\phi = 2$ TeV, the two channels are comparable when $v' \approx 6 \times 10^{-5}$. In the calculation of the branching ratios of ϕ decays, we sum over all six lepton flavor combinations in a flavor-democratic way and we assume

$$Yv' \approx 10^{-10} \text{ GeV} = 0.1 \text{ eV}, \tag{19}$$

so that neutrino masses lie in the expected range. Note that for $v' \approx 6 \times 10^{-5}$ GeV, this implies that $Y \approx 1.6 \times 10^{-6}$. While these couplings are still very small, we consider this parameter freedom to be a strength of our analysis: our approach allows $Y \sim \mathcal{O}(1)$ with a very small v' but at the same time includes the possibility of small Y as well, allowing a large region of parameter space with interesting phenomenology. We also present the branching ratio as a function of the ϕ^{++} mass in Fig. 2(b) for $v' = 6 \times 10^{-5}$ GeV. Here one can see the effect of the different m_ϕ dependence of the $\ell^+\ell^+$ and W^+W^+ final states.

It is interesting to note that the experimental data on neutrino mixing require that at least some of the off-diagonal terms in Y must be of the same order as the diagonal terms when
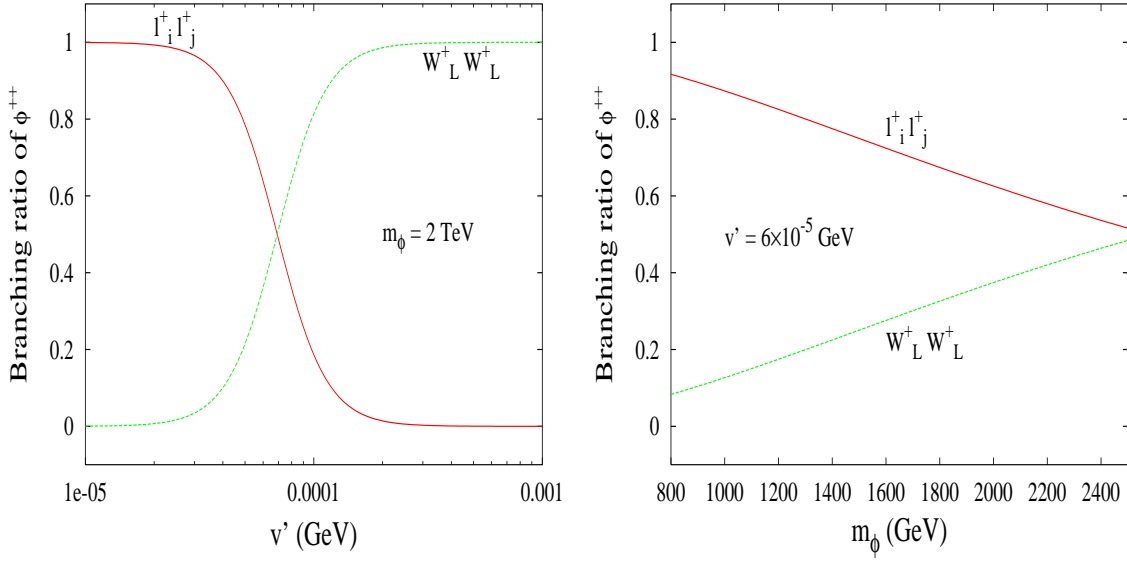
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FIG. 2: Branching ratios of ϕ^{++} (a) versus the triplet vev for $Yv' = 10^{-10}$ GeV and $m_\phi = 2$ TeV and (b) versus m_ϕ for $v' = 6 \times 10^{-5}$ GeV.

written in the charged lepton mass basis. Although the details of the structure depend on the particular neutrino mass matrix, one can, assuming something like a flavor-democratic scenario, immediately envision flavor violating decays such as $\phi^{\pm\pm} \to e^\pm \mu^\pm, \mu^\pm \tau^\pm$ of sizable strength. Such lepton flavor violating decays are a striking signal of this scanerio, where events with two like-sign different-flavor leptons can be observed in a decay final state which reconstructs to an invariant mass peak at m_ϕ.

The branching ratios of ϕ^+ and ϕ^0 receive additional contributions from decays to heavy quarks. Of course, an SU(2) triplet has no dimension-four couplings to quarks. However, in the LtH model such couplings arise from (i) mixing between the triplet and the SU(2) doublet Higgs at order v'/v, and (ii) a dimension-five operator involving both H and ϕ that arises from the expansion of the nonlinear sigma field in the top quark Yukawa Lagrangian, Eq. (A8); inserting the H vev, this yields couplings of ϕ to heavy quark pairs suppressed by v/f. Both of these contributions to the ϕ couplings to heavy quarks are controlled by the relevant Yukawa couplings, m_q/v. The two contributions, proportional to v'/v and v/f respectively, can be seen in the couplings given in Table III.

We are interested in the parameter region $v/f \gg v'/v$, in which case the couplings of ϕ^+ and ϕ^0 to heavy quarks are dominated by the dimension-five nonlinear sigma model operators, yielding an interesting signal of the little Higgs structure in the top sector of the
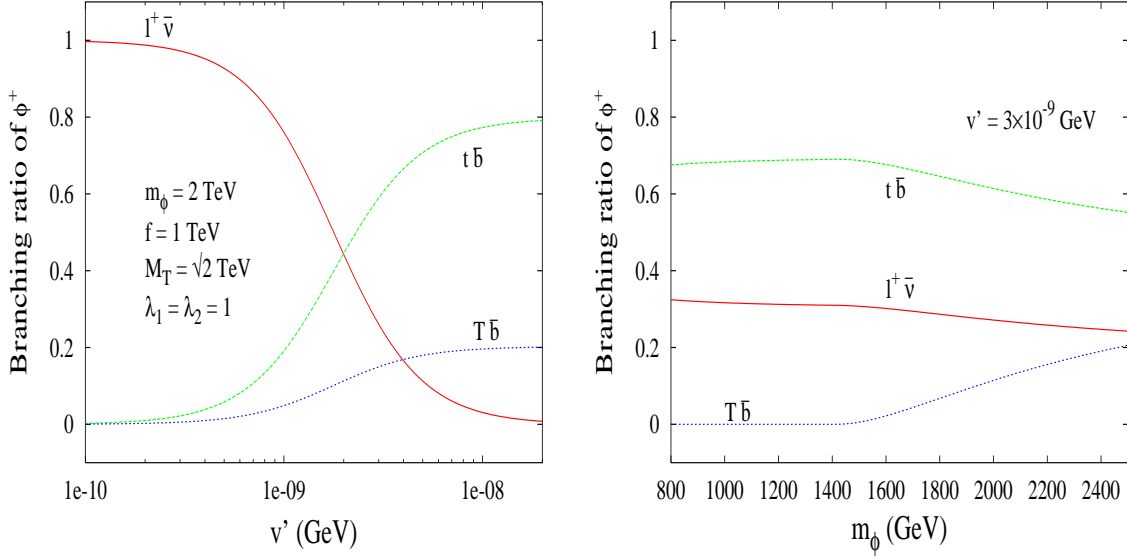
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FIG. 3: Branching ratio of ϕ^+ (a) versus the triplet vev for $Yv' = 10^{-10}$ GeV and $m_\phi = 2$ TeV and (b) versus m_ϕ for $v' = 3 \times 10^{-9}$ GeV.

model. Neglecting final-state masses, the partial decay widths are

$$\Gamma(\phi^+ \to \ell_i^+ \bar\nu_j) = \frac{|Y_{ij}|^2 m_\phi}{8\pi}, \quad \Gamma(\phi^\pm \to t\bar b, \bar t b) \approx \Gamma(\phi^s \to t\bar t) \approx \Gamma(\phi^p \to t\bar t) \approx \frac{N_c m_t^2}{16\pi f^2} m_\phi, \quad (20)$$

where $N_c = 3$ is the number of colors. The triplet couplings to $T\bar b$ and $T\bar t$ also involve the top sector parameters λ_1 and λ_2 (see Appendix A for details) and the decay widths are proportional to $(\lambda_1/\lambda_2)^2$. We illustrate our results for $\lambda_1 = \lambda_2$. Exact formulae for the partial widths are given in Appendix B. Figures 3 and 4 show that the decays of ϕ^+ and ϕ^0 are dominated, approximately from $v' = 2 \times 10^{-9}$ GeV upwards, by the heavy quark final states.

Note that we have treated the triplet mass as a free parameter because of the arbitrary constants a and a' in the coefficient of the triplet mass-squared, as explained in Appendix A. On the other hand, M_T is proportional to f for fixed λ_1, λ_2. Therefore a large value of f in our approach, while the free parameter m_ϕ is held fixed, will suppress the decays into the T-quark. Our results are presented for $M_T = \sqrt{2}$ TeV.

For ϕ^+, the most interesting parameter range is where the elements of Y range between 0.1 and 1, or equivalently v' lies between 10^{-9} and 10^{-10} GeV. In this case ϕ^+ decays mostly into SM leptons, with branching fractions controlled by the structure of the Y_{ij} matrix, which of course directly controls the neutrino masses and mixings. The signatures of ϕ^+
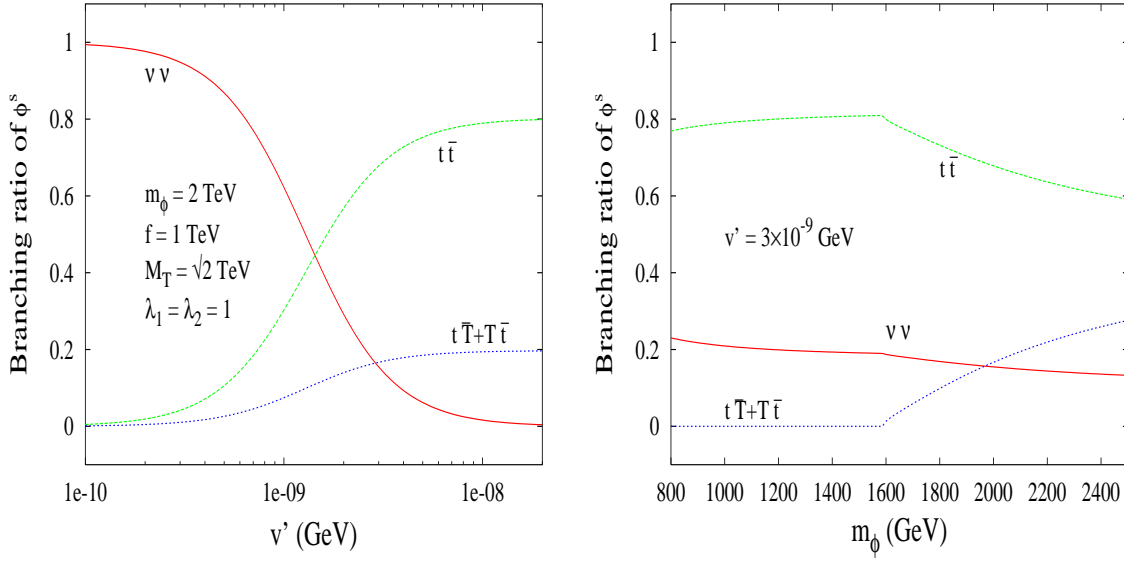
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FIG. 4: Branching ratios of ϕ^s (a) versus the triplet vev for $Yv' = 10^{-10}$ GeV and $m_\phi = 2$ TeV, and (b) versus m_ϕ for $v' = 3 \times 10^{-9}$ GeV. The branching ratios of ϕ^p are virtually identical for the parameter ranges shown.

would then be quite distinct from those of a charged scalar coming from a two-Higgs-doublet model, such as in supersymmetric theories, in which the charged Higgs couplings to leptons are directly proportional to the charged lepton masses. It should also be remembered that this region, with $Y \sim \mathcal{O}(1)$, corresponds to the least number of fine-tuned parameters in the theory. For larger values of v', however, the decays of ϕ^+ will be dominated by the heavy quark final states $t\bar{b}$ (and $T\bar{b}$, if kinematically allowed) which are difficult to distinguish from the decays of the charged Higgs of a two-Higgs-doublet model. For v' below 10^{-4} GeV, the most distinct signals of the triplet will be the $\phi^{\pm\pm}$ decays directly into like-sign dileptons. It should be noted that the $\phi^{\pm\pm}$ does not have any hadronic decay modes to compete with the $\Delta L = 2$ decays in this range of parameters. For larger values of v', the most distinct signals of the triplet will come from $\phi^{\pm\pm} \to W^\pm W^\pm$, giving rise to like-sign dileptons from the W decays which can be identified with suitable event selection criteria.

In the same spirit, the neutral triplet states ϕ^s, ϕ^p are characterized by their invisble decays into two neutrinos for $Y \gtrsim 0.1$, or equivalently $v' \lesssim 10^{-9}$, as shown in Fig. 4 for ϕ^s. The branching ratios of ϕ^p are virtually identical in this parameter range. This makes the neutral scalar ϕ^s and the pseudoscalar ϕ^p quite different in appearance from their counterparts in either the SM or a two-Higgs-doublet model. Such invisible decays can lead

to a detection of the neutral triplet through missing energy signatures or the identification of an invisible state recoiling against a Z boson at a high-energy linear e^+e^- collider.

For the $\phi^\pm$, ϕ^s and ϕ^p, the additional decay modes $\phi^\pm \to W^\pm h$, $\phi^s \to hh$, $\phi^p \to Zh$ are available with the same strength as the $W^\pm Z$ and ZZ modes. However, all these channels are suppressed by v'/v, and they do not stand a chance against either the heavy quark final states or the $\Delta L = 2$ modes. Therefore, the production of the SM Higgs from triplet decays will be unobservable in this scenario.

It should be noted that the region of the parameter space that gives rise to these interesting signals involving leptons will not be accessible in the scenario described in Sec. II D and Refs. [16, 17], in which the $LL\phi$ operator is related to the dimension-five $(LH)^2$ operator through the non-linear sigma model field. Thus the decays of the triplet states can serve to distinguish between alternative scenarios for neutrino mass generation in the LtH model.

A final comment about the decay length of the triplets is in order here. In the region where the $\ell^\pm \ell^\pm$ channel dominates, the lifetime τ of ϕ^{++} (with all flavours summed over) is given by

$$\tau = \frac{8\pi}{9} \frac{v'^2}{(Y_{ij}v')^2} \left(\frac{1 \text{ TeV}}{m_\phi}\right) \times 6.6 \times 10^{-28} \text{ sec.} \tag{21}$$

For $Y_{ij} \approx 1.6 \times 10^{-6}$ (or $v' \approx 6 \times 10^{-5}$ GeV), one finds $\tau \simeq 2.2 \times 10^{-16}$ sec for $m_\phi = 2$ TeV. This gives a decay length $\ell_d \lesssim 0.1$ μm, which is too short to show up as a displaced vertex in the decay. Taking a larger value for v' suppresses the partial width into like-sign lepton pairs, but the WW mode then grows quickly and the decay length remains small.

IV. SUMMARY AND CONCLUSIONS

We have considered the simplest possible scenario for generating the neutrino masses within the context of the Littlest Higgs model by coupling the scalar triplet present in the model to the leptons in a $\Delta L = 2$ interaction. This term then generates neutrino masses through the triplet vev. Although this term does not obey the overseeing $[\text{SU}(2) \times \text{U}(1)]^2$ gauge invariance, it does not affect the cancellation of quadratic divergences in the Higgs mass. We also showed that all contributions coming from dimension-five operators remain subdominant so long as one assumes that there is no lepton-number violating new physics at the scale Λ. Following the phenomenological requirement of keeping the neutrino masses in the required range, we are led to a situation where either the lepton number violating

Yukawa coupling or the triplet vev has to be very small. The second possibility, presumbaly triggered by some yet-unknown feature of the Coleman-Weinberg effective potential, allows one to retain the lepton number violating couplings to be $\mathcal{O}(1)$, a situation that seems less fine-tuned from the viewpoint of allowing bi-large mixing in the neutrino sector.

We have also investigated the decays of the triplet scalar states in this scenario and identified their characteristic features associated with lepton number violation. The most striking signature comes from the doubly-charged scalar decays. The crucial test is the complementarity between the final states of $W^{\pm}W^{\pm}$ and $\ell^{\pm}\ell^{\pm}$: While the triplet vev controls the $W^{\pm}W^{\pm}$ mode and thus the final state branching ratios over a large range, the region corresponding to $Y \approx 1$ leads to significant $\Delta L = 2$ modes, with possibly large lepton-flavor violation. Different complementarity exists for the other triplet scalar decays: between SM heavy quarks (independent of v') and the $\Delta L = 2$ modes. Moreover, the singly-charged scalar may decay to charged leptons with nearly universal couplings, unlike the charged Higgs in typical two-Higgs-doublet models. Another interesting consequence is the "invisible" decay of the neutral triplet state into two neutrinos. These decays would allow one to distinguish models of lepton flavor violation within the Littlest Higgs scenario and directly constrain the elements of the $\Delta L = 2$ coupling matrix which controls the neutrino masses and mixings.

APPENDIX A: THE LITTLEST HIGGS MODEL

1.   Brief summary of the LtH model

The little Higgs approach conceives the Higgs boson as member of a set of pseudo-Goldstone bosons. In the original version of the Littlest Higgs (LtH) scenario [2] to be

discussed here, the pseudo-Goldstone bosons arise when a global SU(5) symmetry is broken down to SO(5) at a scale $\Lambda \sim 4\pi f$. These pseudo-Goldstone bosons are described by a nonlinear sigma model below the scale Λ.

The breakdown of the global symmetry is triggered by a vacuum expectation value (vev) Σ_0 of the sigma-model field,

$$\Sigma = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f}, \tag{A1}$$

where $\Pi = \sum_a \Pi^a X^a$ and X^a correspond to the 14 broken SU(5) generators. Explicitly, we have

$$\Sigma_0 = \begin{pmatrix} & & \mathbf{1}_{2\times2} \\ & 1 & \\ \mathbf{1}_{2\times2} & & \end{pmatrix}, \qquad \Pi = \begin{pmatrix} \mathbf{0}_{2\times2} & \frac{H^\dagger}{\sqrt{2}} & \phi^\dagger \\ \frac{H^*}{\sqrt{2}} & 0 & \frac{H}{\sqrt{2}} \\ \phi & \frac{H^T}{\sqrt{2}} & \mathbf{0}_{2\times2} \end{pmatrix}, \tag{A2}$$

where we have suppressed the Goldstone modes that will later be eaten by broken gauge generators, and we define

$$H = (h^+, h^0), \qquad \phi = -i\begin{pmatrix} \phi^{++} & \frac{\phi^+}{\sqrt{2}} \\ \frac{\phi^+}{\sqrt{2}} & \phi^0 \end{pmatrix}. \tag{A3}$$

An $[SU(2)\times U(1)]^2$ subgroup of the global SU(5) is gauged. The Σ_0 vev that is responsible for the breakdown of the global symmetry also breaks the gauged $[SU(2)\times U(1)]^2$ down to the SM electroweak gauge symmetry $SU(2)_L\times U(1)_Y$. Under the electroweak gauge group, H and ϕ transform as a complex doublet and a complex triplet, respectively.

The gauge interaction of the sigma field is encoded in its covariant derivative:

$$\mathcal{L}_\Sigma = \frac{f^2}{8}Tr|D_\mu\Sigma|^2, \tag{A4}$$

where

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_{j=1,2}[g_j W_{j\mu}^a(Q_j^a\Sigma + \Sigma Q_j^{aT}) + g_j' B_{j\mu}(Y_j\Sigma + \Sigma Y_j^T)]. \tag{A5}$$

Here Q_j^a are the SU(2) generators and Y_j are the U(1) generators, which explicitly break the global SU(5) symmetry:

$$Q_1^a = \begin{pmatrix} \frac{\sigma^a}{2} & \\ & \mathbf{0}_{3\times3} \end{pmatrix}, \qquad Q_2^a = \begin{pmatrix} \mathbf{0}_{3\times3} & \\ & \frac{\sigma^{a*}}{2} \end{pmatrix}, \tag{A6}$$

$$Y_1 = \frac{1}{10}\mathrm{diag}(-3,-3,2,2,2), \qquad Y_2 = \frac{1}{10}\mathrm{diag}(-2,-2,-2,3,3). \tag{A7}$$

Notice that setting $g_1 = g_1' = 0$ leaves unbroken an SU(3) subgroup of the global SU(5) symmetry; we call this remaining global symmetry SU(3)$_1$. Similarly, setting $g_2 = g_2' = 0$ leaves unbroken a second SU(3) subgroup of the global SU(5) symmetry, which we call SU(3)$_2$. The Higgs doublet H transforms nonlinearly under both of these global SU(3) symmetries, and thus remains an exact Goldstone boson so long as these global symmetries are not explicitly broken. A Higgs mass term can thus be generated only by interactions involving both g_1 and g_2 (or both g_1' and g_2'); this serves to forbid the diagrams that generate the quadratic divergence in the Higgs mass at one loop. However, logarithmically divergent diagrams contributing to the Higgs mass at one loop involve both gauge couplings g_1 and g_2 (or both g_1' and g_2') and thus break the global SU(3), thereby leading to contributions to the Higgs mass.

In order to cancel the quadratic divergence arising through the top quark Yukawa coupling, we have to introduce a heavy vector-like quark pair (T, T^c), where T is left-handed and has charge $+2/3$. Including this vectorlike pair, the top Yukawa Lagrangian is

$$\mathcal{L}_t = \frac{\lambda_1}{2} f \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} t^c + \lambda_2 f T T^c + \text{h.c.}, \tag{A8}$$

where $\chi^T = (b_L, t_L, T)$ and t^c is an SU(2) singlet. The indices i, j, k take the values 1,2,3, whereas x, y take the values 4,5. It should be noted here that the coupling λ_1 preserves the global SU(3)$_1$ and breaks SU(3)$_2$, while λ_2 preserves SU(3)$_2$ and breaks SU(3)$_1$. This ensures that the Higgs mass-squared is protected from quadratic divergences involving the top quark sector at one loop. Diagonalizing the mass matrix arising from Eq. (A8), we find the physical top quark t and a heavy isospin-singlet "top-partner" T:

$$m_t \simeq \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} v, \qquad M_T \simeq f \sqrt{\lambda_1^2 + \lambda_2^2}. \tag{A9}$$

The gauge and top quark interactions generate a Higgs potential at one loop via the Coleman-Weinberg mechanism [14], which is given by

$$V_{CW} = \lambda_{\phi^2} f^2 \text{Tr}(\phi^\dagger \phi) + i\lambda_{h\phi h} f(H\phi^\dagger H^T - H^* \phi H^\dagger) - \mu^2 HH^\dagger + \lambda_{h^4}(HH^\dagger)^2$$

$$+ \lambda_{h\phi\phi h} H\phi^\dagger \phi H^\dagger + \lambda_{h^2\phi^2} HH^\dagger \text{Tr}(\phi^\dagger \phi) + \lambda_{\phi^2\phi^2}[\text{Tr}(\phi^\dagger\phi)]^2 + \lambda_{\phi^4} \text{Tr}(\phi^\dagger \phi \phi^\dagger \phi), \tag{A10}$$

with coefficients

$$\lambda_{\phi^2} = \frac{a}{2}\left[\frac{g^2}{s^2 c^2} + \frac{g'^2}{s'^2 c'^2}\right] + 8a'\lambda_1^2 \tag{A11}$$

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$$\lambda_{h\phi h} = -\frac{a}{4}\left[g^2\frac{(c^2-s^2)}{s^2c^2} + g'^2\frac{(c'^2-s'^2)}{s'^2c'^2}\right] + 4a'\lambda_1^2 \tag{A12}$$

$$\lambda_{h^4} = \frac{1}{4}\lambda_{\phi^2}, \qquad \lambda_{h\phi\phi h} = -\frac{4}{3}\lambda_{\phi^2}, \qquad \lambda_{\phi^2\phi^2} = -16a'\lambda_1^2 \tag{A13}$$

$$\lambda_{\phi^4} = -\frac{2a}{3}\left[\frac{g^2}{s^2c^2} + \frac{g'^2}{s'^2c'^2}\right] + \frac{16a'}{3}\lambda_1^2. \tag{A14}$$

where c and s (c' and s') are the gauge coupling mixing parameters for the SU(2) (U(1)) gauge groups, respectively [15]. Here a, a' are parameters of $\mathcal{O}(1)$ that encapsulate the cutoff dependence of the gauge and top sectors, respectively, of the UV-incomplete theory. The parameters μ^2 and $\lambda_{h^2\phi^2}$ are generated through logarithmic contributions. Electroweak symmetry breaking is triggered if $\mu^2 > 0$, whereby the scalar doublet acquires a vev. The triplet vev is kept small by keeping λ_{ϕ^2} positive; it originates in mixing with the doublet via $\lambda_{h\phi h}$. The minimization conditions for V_{CW}, in terms of $\langle h^0 \rangle = v/\sqrt{2}$, $\langle \phi^0 \rangle = v'$, are

$$v^2 = \frac{\mu^2}{\lambda_{h^4} - \frac{\lambda_{h\phi h}^2}{\lambda_{\phi^2}}}, \qquad v' = \frac{\lambda_{h\phi h}v^2}{2\lambda_{\phi^2}f}. \tag{A15}$$

Note that terms of the form $H^2\phi^2, \phi^4$ give a subleading contribution to Eq. (A15) and have been neglected. In order to ensure electroweak symmetry breaking, we should have $\lambda_{h^4} - \frac{\lambda_{h\phi h}^2}{\lambda_{\phi^2}} > 0$. The resulting masses for the triplet states ϕ and the physical Higgs boson h after electroweak symmetry breaking are

$$m_\phi^2 \simeq \lambda_{\phi^2}f^2, \qquad m_h^2 \simeq 2\left(\lambda_{h^4} - \frac{\lambda_{h\phi h}^2}{\lambda_{\phi^2}}\right)v^2 \simeq 2\mu^2. \tag{A16}$$

It should also be noted that λ_{ϕ^2}, as expressed above, gets modified by an additional term once $\Delta L = 2$ interactions are switched on, as has been shown in Sec. II B.

2. Lepton number violation

When we introduce the $\Delta L = 2$ interaction of Eq. (3) in order to give rise to neutrino masses, one of its effects is to add an extra term to the expression of Eq. (A11) for λ_{ϕ^2}, as shown in Eq. (5). This contribution is typically small in the parameter ranges that we consider.

As for the $\Delta L = 2$ interactions of the triplet ϕ, expanding Eq. (3) explicitly one can obtain the full lepton number violating interaction vertices. The dimension-four couplings are given in Eq. (15). The Feynman rules for the $\Delta L = 2$ interactions are given in Table II. The

$\phi^{--}\ell_i^+\ell_j^+$ $(i \le j)$	$2iY_{ij}^* P_R C$
$\phi^- \ell_i^+ \bar{\nu}_j$	$i\sqrt{2}Y_{ij}^* P_R C$
$\phi^s \nu_i \nu_j$ $(i \le j)$	$i\sqrt{2}Y_{ij} C^{-1} P_L$
$\phi^s \bar{\nu}_i \bar{\nu}_j$ $(i \le j)$	$i\sqrt{2}Y_{ij}^* P_R C$
$\phi^p \nu_i \nu_j$ $(i \le j)$	$-\sqrt{2}Y_{ij} C^{-1} P_L$
$\phi^p \bar{\nu}_i \bar{\nu}_j$ $(i \le j)$	$\sqrt{2}Y_{ij}^* P_R C$

TABLE II: Feynman rules for $\Delta L = 2$ couplings. All particles and momenta are outgoing. C is the charge-conjugation operator. Since Y_{ij} is symmetric under (i,j) we have combined the symmetric vertices involving ϕ^{--}, ϕ^s and ϕ^p and written them only for $i \le j$.

relevant lepton number conserving interactions between the triplet state and SM particles [15] are given as Feynman rules in Table III. For the $\phi^s hh$ coupling, we have included the symmetry factor, Feynman rule $= i\mathcal{L} \times 2$, and used the relation in Eq. (A15) to write $\lambda_{h\phi h}$ in terms of v'.

APPENDIX B: TRIPLET DECAY PARTIAL WIDTHS

In this Appendix we present the formulas for the triplet decay partial widths. We define the standard kinematic function $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ and use the scaled mass variable $r_i = m_i/m_\phi$. For the doubly-charged scalar ϕ^{++}, we have

$$
\Gamma(\phi^{++} \to \ell_i^+ \ell_j^+) = \begin{cases} \frac{1}{8\pi}|Y_{ij}|^2 m_\phi, & (i = j) \\ \frac{1}{4\pi}|Y_{ij}|^2 m_\phi, & (i < j) \end{cases}
$$

$$
\Gamma(\phi^{++} \to W_T^+ W_T^+) = \frac{1}{4\pi} \frac{g^4 v'^2}{m_\phi} \frac{\lambda^{\frac{1}{2}}(1, r_W^2, r_W^2)}{\sqrt{4r_W^2 + \lambda(1, r_W^2, r_W^2)}} \approx \frac{g^4 v'^2}{4\pi m_\phi},
$$

$$
\Gamma(\phi^{++} \to W_L^+ W_L^+) = \frac{1}{4\pi} \frac{g^4 v'^2}{2m_\phi} \frac{\lambda^{\frac{1}{2}}(1, r_W^2, r_W^2)}{\sqrt{4r_W^2 + \lambda(1, r_W^2, r_W^2)}} \frac{(1 - 4r_W^2)^2}{4r_W^4} \approx \frac{v'^2 m_\phi^3}{2\pi v^4}, \quad \text{(B1)}
$$

where in the last two expressions we have shown the approximate result neglecting final-state masses compared to m_ϕ. We use the subscripts T and L to denote the transverse and longitudinal polarizations of the SM gauge bosons.

$\phi^{--}W_\mu^+W_\nu^+$	$2ig^2v'g_{\mu\nu}$
$\phi^-W_\mu^+Z_\nu$	$-i\frac{g^2}{c_W}v'g_{\mu\nu}$
$\phi^-W_\mu^+h$	$-ig\frac{v'}{v}(p_h-p_\phi)_\mu$
$\phi^-\bar{b}t$	$-\frac{i}{\sqrt{2}v}(m_tP_R+m_bP_L)(\frac{v}{f}-4\frac{v'}{v})$
$\phi^-\bar{b}T$	$-\frac{im_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\frac{\lambda_1}{\lambda_2}P_R$
$\phi^sZ_\mu Z_\nu$	$i\sqrt{2}\frac{g^2}{c_W^2}v'g_{\mu\nu}$
ϕ^shh	$i2\sqrt{2}m_\phi^2\frac{v'}{v^2}$
$\phi^sW_\mu^+W_\nu^-$	0
$\phi^s\bar{t}t$	$-\frac{im_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})$
$\phi^s\bar{b}b$	$-\frac{im_b}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})$
$\phi^s\bar{t}T$	$-\frac{im_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\frac{\lambda_1}{\lambda_2}P_R$
$\phi^s\bar{T}t$	$-\frac{im_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\frac{\lambda_1}{\lambda_2}P_L$
$\phi^pZ_\mu h$	$-\sqrt{2}\frac{g}{c_W}\frac{v'}{v}(p_h-p_\phi)_\mu$
$\phi^p\bar{t}t$	$-\frac{m_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\gamma^5$
$\phi^p\bar{b}b$	$\frac{m_b}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\gamma^5$
$\phi^p\bar{t}T$	$\frac{m_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\frac{\lambda_1}{\lambda_2}P_R$
$\phi^p\bar{T}t$	$\frac{m_t}{\sqrt{2}v}(\frac{v}{f}-4\frac{v'}{v})\frac{\lambda_1}{\lambda_2}P_L$

TABLE III: Feynman rules for lepton number conserving ϕ couplings to SM particles, from Ref. [15]. All particles and momenta are outgoing.

For the singly-charged scalar ϕ^+, we have,

$$\Gamma(\phi^+\to\ell_i^+\bar{\nu}_j)=\frac{1}{8\pi}|Y_{ij}|^2m_\phi,$$

$$\Gamma(\phi^+\to W_T^+Z_T)=\frac{1}{4\pi}\frac{g^4v'^2}{m_\phi c_W^2}\left[\frac{\lambda^{\frac{1}{2}}(1,r_W^2,r_Z^2)}{\sqrt{4r_W^2+\lambda(1,r_W^2,r_Z^2)}+\sqrt{4r_Z^2+\lambda(1,r_W^2,r_Z^2)}}\right]$$

$$\approx\frac{g^4v'^2}{8\pi m_\phi c_W^2},$$

$$\Gamma(\phi^+\to W_L^+h)=\frac{1}{4\pi}\frac{g^2v'^2}{v^2}\frac{m_\phi}{2r_W^2}\left[\frac{\lambda^{\frac{3}{2}}(1,r_h^2,r_W^2)}{\sqrt{4r_h^2+\lambda(1,r_h^2,r_W^2)}+\sqrt{4r_W^2+\lambda(1,r_h^2,r_W^2)}}\right]$$

$$\approx\frac{v'^2m_\phi^3}{4\pi v^4}$$

$$\Gamma(\phi^+ \to W_L^+ Z_L) = \frac{1}{4\pi} \frac{g^4 v'^2}{2m_\phi c_W^2} \left[\frac{\lambda^{\frac{1}{2}}(1, r_W^2, r_Z^2)}{\sqrt{4r_W^2 + \lambda(1, r_W^2, r_Z^2)} + \sqrt{4r_Z^2 + \lambda(1, r_W^2, r_Z^2)}} \right]$$

$$\times \frac{(1 - r_W^2 - r_Z^2)^2}{4r_W^2 r_Z^2} \approx \frac{v'^2 m_\phi^3}{4\pi v^4}$$

$$\Gamma(\phi^+ \to t\bar{b}) = \frac{N_c}{4\pi} \frac{r_t^2 m_\phi^3}{4f^2} \left[\frac{\lambda^{\frac{1}{2}}(1, r_t^2, r_b^2)(1 - r_t^2 - r_b^2)}{\sqrt{4r_t^2 + \lambda(1, r_t^2, r_b^2)} + \sqrt{4r_b^2 + \lambda(1, r_t^2, r_b^2)}} \right] \approx \frac{N_c m_t^2 m_\phi}{32\pi f^2},$$

$$\Gamma(\phi^+ \to T\bar{b}) = \frac{N_c}{4\pi} \frac{r_t^2 m_\phi^3}{4f^2} \left[\frac{\lambda^{\frac{1}{2}}(1, r_T^2, r_b^2)(1 - r_T^2 - r_b^2)}{\sqrt{4r_T^2 + \lambda(1, r_T^2, r_b^2)} + \sqrt{4r_b^2 + \lambda(1, r_T^2, r_b^2)}} \right] \left(\frac{\lambda_1}{\lambda_2} \right)^2$$

$$\approx \frac{N_c m_t^2 m_\phi}{32\pi f^2} \left(\frac{\lambda_1}{\lambda_2} \right)^2 (1 - r_T^2)^2. \tag{B2}$$

For the neutral scalar ϕ^s, we have

$$\Gamma(\phi^s \to \nu_i \nu_j + \bar{\nu}_i \bar{\nu}_j) = \begin{cases} \frac{1}{8\pi} |Y_{ij}|^2 m_\phi, & (i = j) \\ \frac{1}{4\pi} |Y_{ij}|^2 m_\phi, & (i < j) \end{cases}$$

$$\Gamma(\phi^s \to Z_T Z_T) = \frac{1}{4\pi} \frac{g^4 v'^2}{2m_\phi c_W^4} \frac{\lambda^{\frac{1}{2}}(1, r_Z^2, r_Z^2)}{\sqrt{4r_Z^2 + \lambda(1, r_Z^2, r_Z^2)}} \approx \frac{g^4 v'^2}{8\pi m_\phi c_W^4}$$

$$\Gamma(\phi^s \to hh) = \frac{1}{4\pi} \frac{v'^2 m_\phi^3}{v^4} \frac{\lambda^{\frac{1}{2}}(1, r_h^2, r_h^2)}{\sqrt{4r_h^2 + \lambda(1, r_h^2, r_h^2)}} \approx \frac{v'^2 m_\phi^3}{4\pi v^4}$$

$$\Gamma(\phi^s \to Z_L Z_L) = \frac{1}{4\pi} \frac{g^4 v'^2}{4m_\phi c_W^4} \frac{\lambda^{\frac{1}{2}}(1, r_Z^2, r_Z^2)}{\sqrt{4r_Z^2 + \lambda(1, r_Z^2, r_Z^2)}} \frac{(1 - 4r_Z^2)^2}{4r_Z^4} \approx \frac{v'^2 m_\phi^3}{4\pi v^4}$$

$$\Gamma(\phi^s \to t\bar{t}) = \frac{N_c}{4\pi} \frac{r_t^2 m_\phi^3}{4f^2} \frac{\lambda^{\frac{1}{2}}(1, r_t^2, r_t^2)}{\sqrt{4r_t^2 + \lambda(1, r_t^2, r_t^2)}} (1 - 4r_t^2) \approx \frac{N_c m_t^2 m_\phi}{16\pi f^2},$$

$$\Gamma(\phi^s \to b\bar{b}) = \frac{N_c}{4\pi} \frac{r_b^2 m_\phi^3}{4f^2} \frac{\lambda^{\frac{1}{2}}(1, r_b^2, r_b^2)}{\sqrt{4r_b^2 + \lambda(1, r_b^2, r_b^2)}} (1 - 4r_b^2) \approx \frac{N_c m_b^2 m_\phi}{16\pi f^2},$$

$$\Gamma(\phi^s \to T\bar{t} + t\bar{T}) = \frac{N_c}{4\pi} \frac{r_t^2 m_\phi^3}{2f^2} \left[\frac{\lambda^{\frac{1}{2}}(1, r_T^2, r_t^2)(1 - r_T^2 - r_t^2)}{\sqrt{4r_T^2 + \lambda(1, r_T^2, r_t^2)} + \sqrt{4r_t^2 + \lambda(1, r_T^2, r_t^2)}} \right] \left(\frac{\lambda_1}{\lambda_2} \right)^2$$

$$\approx \frac{N_c m_t^2 m_\phi}{16\pi f^2} \left(\frac{\lambda_1}{\lambda_2} \right)^2 (1 - r_T^2)^2. \tag{B3}$$

Finally, for the neutral pseudoscalar ϕ^p, we have

$$\Gamma(\phi^p \to \nu_i \nu_j + \bar{\nu}_i \bar{\nu}_j) = \begin{cases} \frac{1}{8\pi} |Y_{ij}|^2 m_\phi, & (i = j) \\ \frac{1}{4\pi} |Y_{ij}|^2 m_\phi, & (i < j) \end{cases}$$

$$\Gamma(\phi^p \to Z_L h) = \frac{1}{4\pi} \frac{g^2 v'^2 m_\phi}{v^2 c_W^2 r_Z^2} \left[\frac{\lambda^{\frac{3}{2}}(1, r_h^2, r_Z^2)}{\sqrt{4r_h^2 + \lambda(1, r_h^2, r_Z^2)} + \sqrt{4r_Z^2 + \lambda(1, r_h^2, r_Z^2)}} \right]$$

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$$\approx \frac{v'^2 m_\phi^3}{2\pi v^4}$$

$$\Gamma(\phi^p \to t\bar{t}) = \frac{N_c}{4\pi} \frac{r_t^2 m_\phi^3}{4f^2} \frac{\lambda^{\frac{1}{2}}(1, r_t^2, r_t^2)}{\sqrt{4r_t^2 + \lambda(1, r_t^2, r_t^2)}} \approx \frac{N_c m_t^2 m_\phi}{16\pi f^2}$$

$$\Gamma(\phi^p \to b\bar{b}) = \frac{N_c}{4\pi} \frac{r_b^2 m_\phi^3}{4f^2} \frac{\lambda^{\frac{1}{2}}(1, r_b^2, r_b^2)}{\sqrt{4r_b^2 + \lambda(1, r_b^2, r_b^2)}} \approx \frac{N_c m_b^2 m_\phi}{16\pi f^2}$$

$$\Gamma(\phi^p \to T\bar{t} + t\bar{T}) = \frac{N_c}{4\pi} \frac{r_t^2 m_\phi^3}{2f^2} \left[\frac{\lambda^{\frac{1}{2}}(1, r_T^2, r_t^2)(1 - r_T^2 - r_t^2)}{\sqrt{4r_T^2 + \lambda(1, r_T^2, r_t^2)} + \sqrt{4r_t^2 + \lambda(1, r_T^2, r_t^2)}} \right] \left(\frac{\lambda_1}{\lambda_2}\right)^2$$

$$\approx \frac{N_c m_t^2 m_\phi}{16\pi f^2} \left(\frac{\lambda_1}{\lambda_2}\right)^2 (1 - r_T^2)^2. \tag{B4}$$

In the ϕ^+, ϕ^s, ϕ^p couplings to quarks, we have neglected v'/v relative to v/f and included the color factor, $N_c = 3$.

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